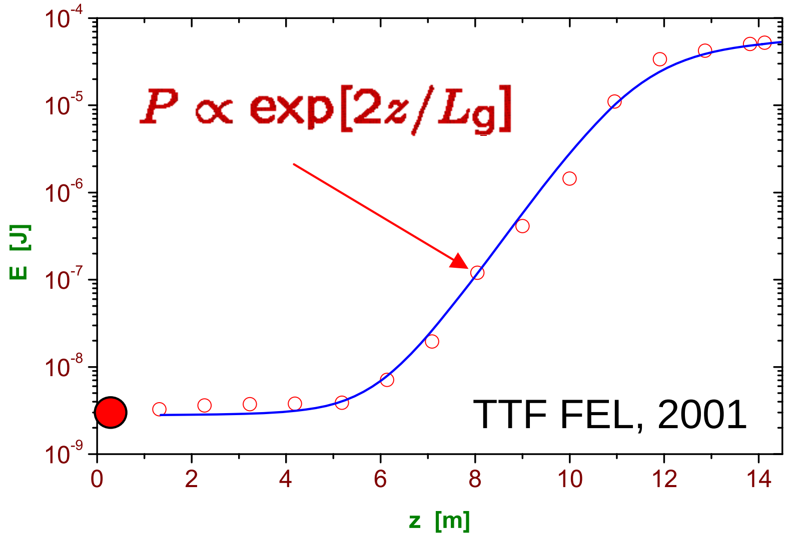
Coherence properties of the radiation from x-ray free electron lasers

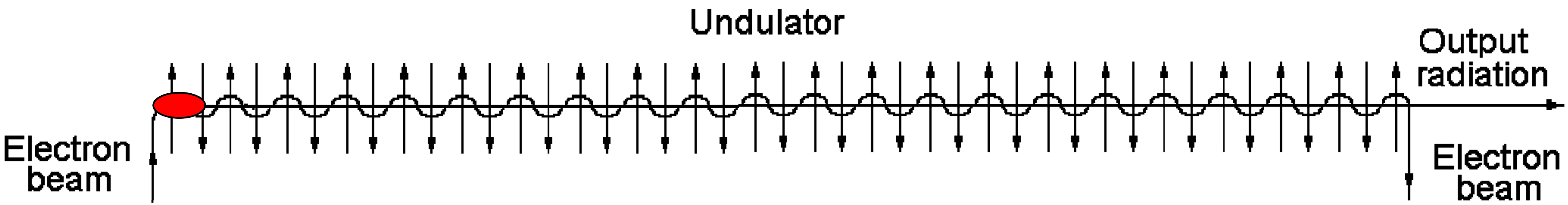
E.A. Schneidmiller, M.V. Yurkov

DESY, Hamburg

- Statistical properties.
- Longitudinal and transverse coherence.
- Higher harmonics.

Qualitative look at the radiation properties

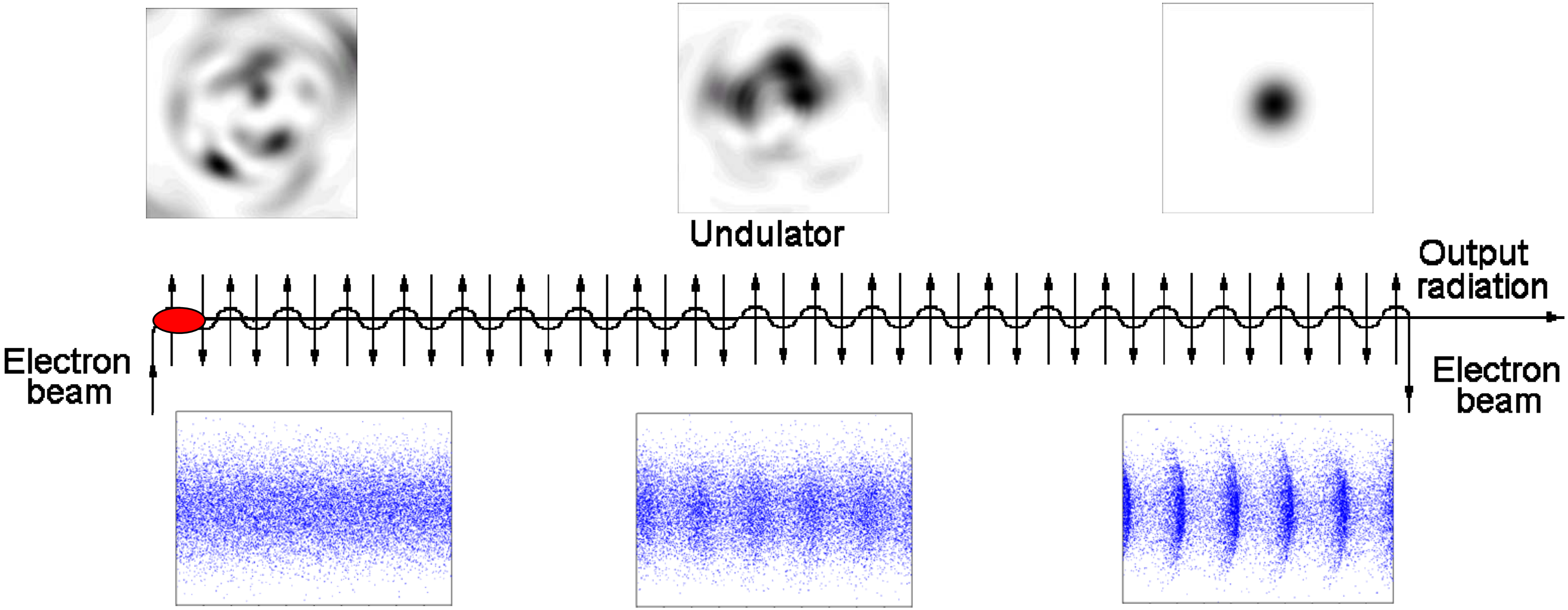




- Self Amplified Spontaneous Emission (SASE) FEL is an attractively simple device: it is just a system consisting of a relativistic electron beam and an undulator only.
- SASE FEL is capable to produce high power and high quality radiation (in terms of coherence properties).

Ya.S. Derbenev, A.M. Kondratenko, and E.L. Saldin, Nucl. nstrum. and Methods 193(1982)415

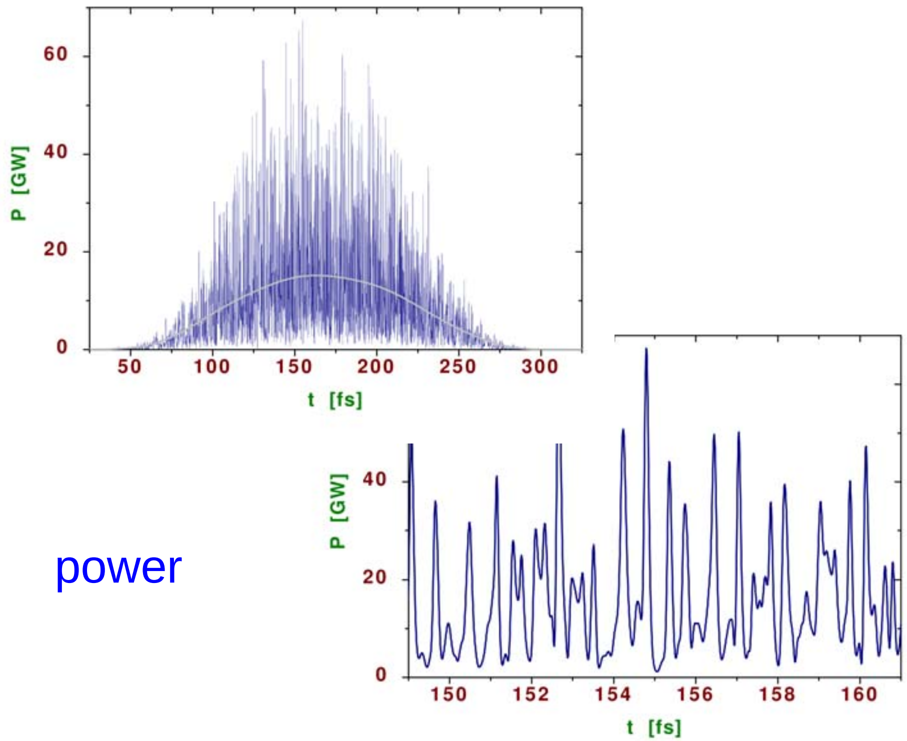
Qualitative look at the radiation properties



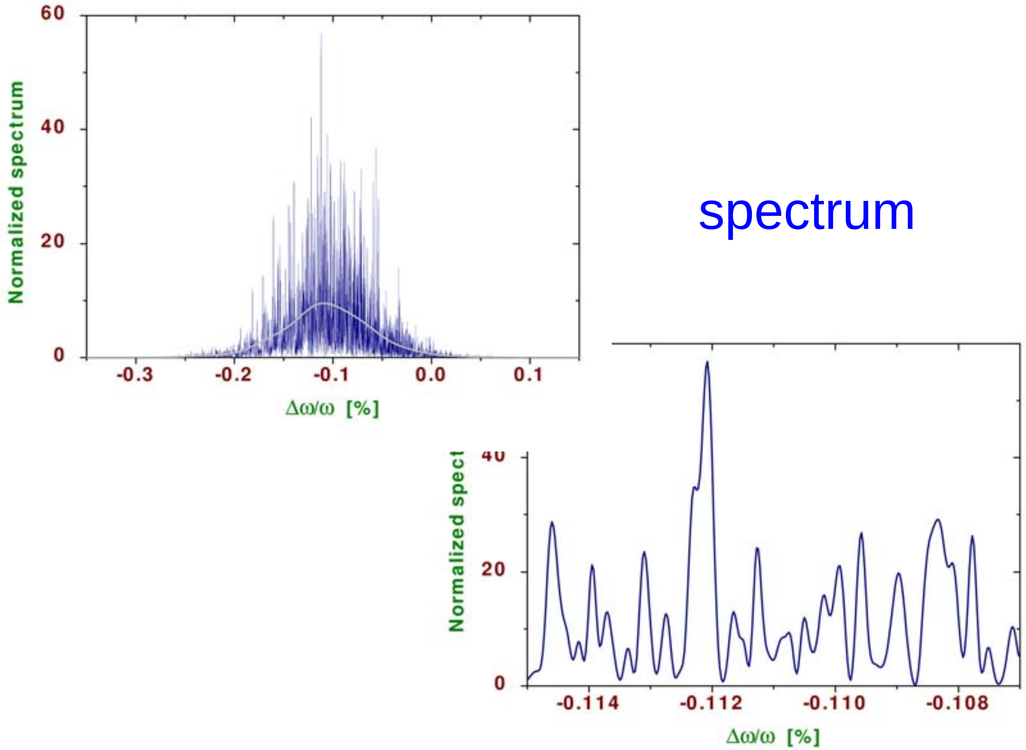
- Longitudinal coherence is formed due to slippage effects (electromagnetic wave advances electron beam by one wavelength while electron beam passes one undulator period). Thus, typical figure of merit is relative slippage of the radiation with respect to the electron beam on a scale of field gain length ➔ coherence time.
- Transverse coherence is formed due to diffraction effects. Typical figure of merit is ratio of the diffraction expansion of the radiation on a scale of field gain length to the transverse size of the electron beam.

Qualitative look at the radiation properties

power



spectrum



- Radiation generated by SASE FEL consists of wavepackets (spikes). Typical duration of the spike is about coherence time τ_c.
- Spectrum also exhibits spiky structure. Spectrum width is inversely proportional to the coherence time, $\Delta\omega$ » $1/\tau_c$, and typical width of a spike in a spectrum is inversely proportional to the pulse duration T.
- Amplification process selects narrow band of the radiation, coherence time is increased, and spectrum is shrinked. Transverse coherence is improved as well due to the mode selection process.

Strict definitions of statistical characteristics

- The first order time correlation function and coherence time:

$$g_1(\vec{r}, t-t') = \frac{\langle \tilde{E}(\vec{r},t)\tilde{E}^*(\vec{r},t')\rangle}{\left[\langle |\ \tilde{E}(\vec{r},t)\ |^2\rangle\langle |\ \tilde{E}(\vec{r},t')\ |^2\rangle\right]^{1/2}} , \qquad \tau_c = \int\limits_{-\infty}^{\infty} |g_1(\tau)|^2 \, \mathrm{d}\,\tau \ .$$

- The first-order transverse correlation function and degree of transverse coherence:

$$\gamma_1(\vec{r}_\perp, \vec{r\prime}_\perp, z, t) = \frac{\langle \tilde{E}(\vec{r}_\perp, z, t)\tilde{E}^*(\vec{r\prime}_\perp, z, t)\rangle}{\left[\langle|\tilde{E}(\vec{r}_\perp, z, t)|^2\rangle\langle|\tilde{E}(\vec{r\prime}_\perp, z, t)|^2\rangle\right]^{1/2}} \ .$$

$$\zeta = \frac{\int\int |\gamma_1(\vec{r}_\perp, \vec{r\prime}_\perp)|^2 \langle I(\vec{r}_\perp)\rangle\langle I(\vec{r\prime}_\perp)\rangle \, \mathrm{d}\,\vec{r}_\perp \, \mathrm{d}\,\vec{r\prime}_\perp}{[\int\langle I(\vec{r}_\perp)\rangle \, \mathrm{d}\,\vec{r}_\perp]^2} \ .$$

- Degeneracy parameter – the number of photons per mode (coherent state):

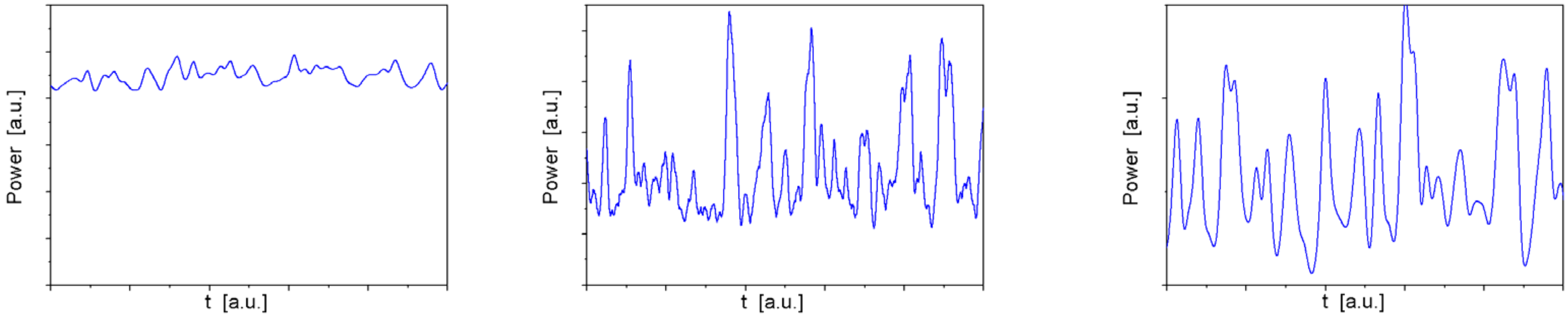
$$\delta = \dot{N}_{ph}\tau_c\zeta \ .$$

- Peak brilliance is defined as a transversely coherent spectral flux:

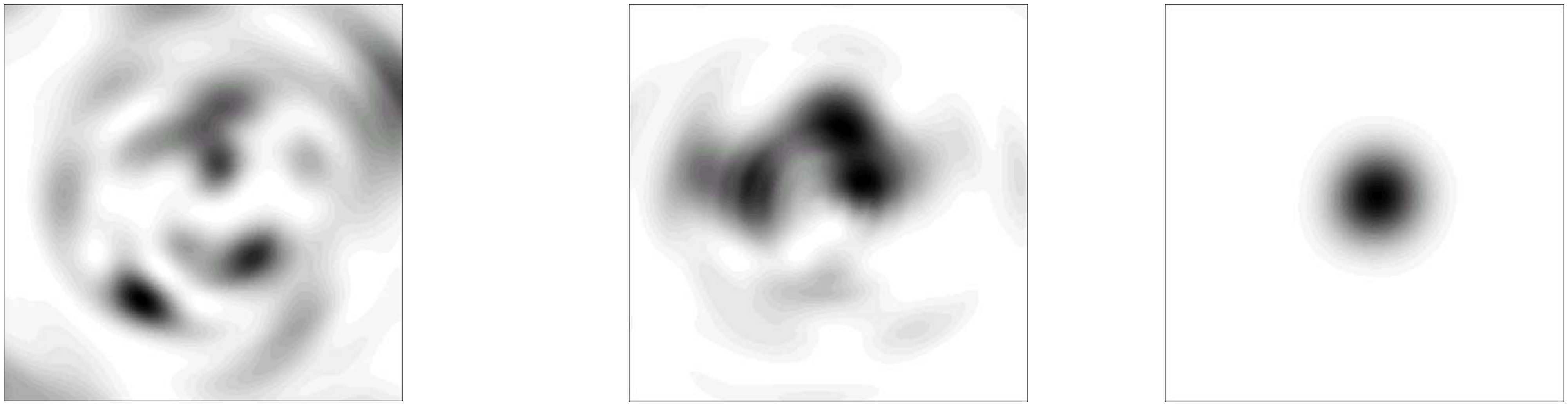
$$B_r = \frac{\omega \, \mathrm{d}\,\dot{N}_{ph}}{\mathrm{d}\,\omega} \frac{\zeta}{\left(\frac{\lambda}{2}\right)^2} = \frac{4\sqrt{2}c}{\lambda^3}\,\delta \ .$$

Statistics and probability distributions

$z = 0.1\ z_{sat}$ | $z = 0.5\ z_{sat}$ | $z = z_{sat}$

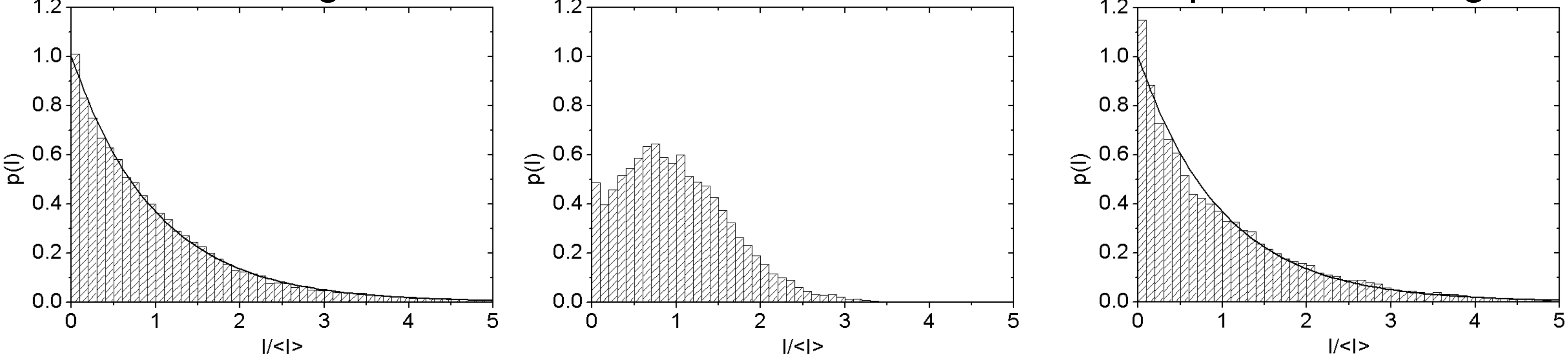


- Transverse (bottom) and longitudinal (top) distributions of the radiation intensity exhibit rather chaotic behaviour.

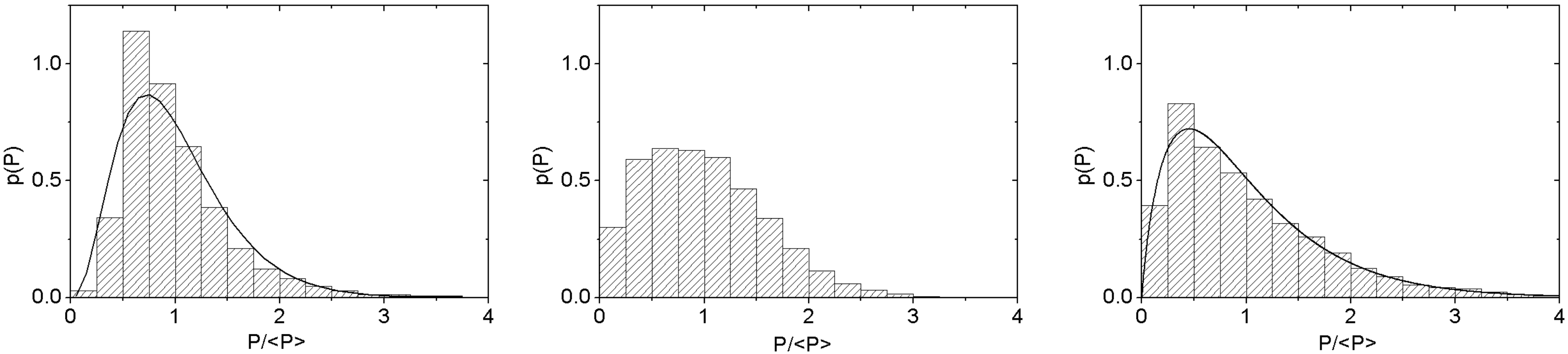


Statistics and probability distributions

Linear regime | Saturation | Deep nonlinear regime



- Probability distributions of the instantaneous power density (top) and of the instantaneous radiation power (bottom) look more elegant and seem to be described by simple functions.



Statistics and probability distributions

- The origin of this fundamental simplicity relates to the properties of the electron beam. The shot noise in the electron beam has a statistical nature that significantly influences characteristics of the output radiation from a SASE FEL.
- Fluctuations of the electron beam current density serve as input signals in a SASE FEL. These fluctuations always exist in the electron beam due to the effect of shot noise. Initially fluctuations are not correlated in space and time, but when the electron beam enters the undulator, beam modulation at frequencies close to the resonance frequency of the FEL amplifier initiates the process of the amplification of coherent radiation.
- Electron beam current is $I(t) = (-e)\,\Sigma_{k=1}^{N}\,\delta(t - t_k)$, and its Fourier harmonic is just a sum of complex phasors:

$$\bar{I}(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} I(t)dt = (-e)\sum_{k=1}^{N} e^{i\omega t_k}$$

- Thus, we deal with gaussian statistical process. FEL amplifier operating in the linear regime is just linear filter, $\bar{E}(\omega) = H_A(\omega - \omega_0)\bar{I}(\omega)$, which does not change statistics.

Statistics and probability distributions

Radiation from SASE FEL operating in the high gain linear regime possesses all the features of completely chaotic polarized light:

- The higher order correlation functions are expressed via the first order correlation function

$$g_2(t-t') = 1 + |g_1(t-t')|^2$$

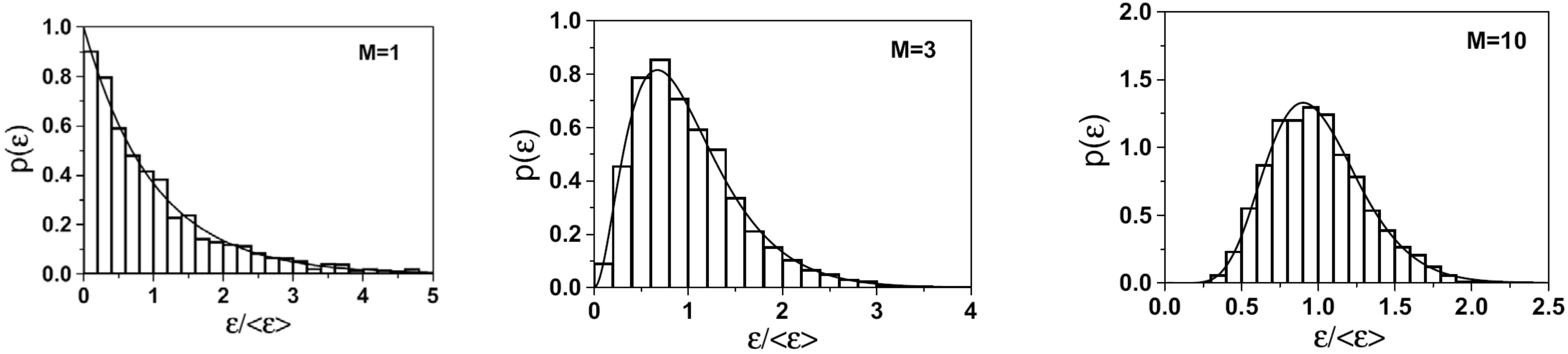
$$g_2(\Delta\omega) = 1 + |g_1(\Delta\omega)|^2$$

- The probability density distribution of the instantaneous radiation power follows the negative exponential distribution

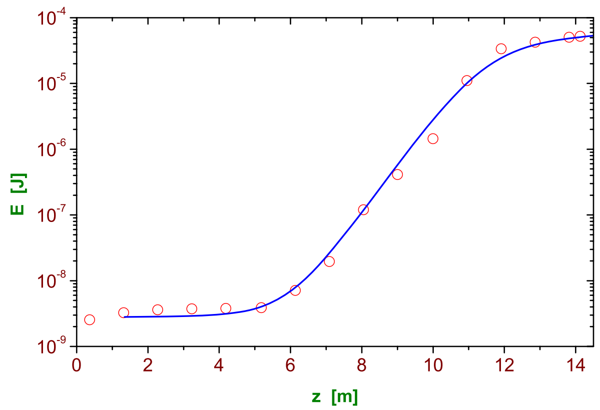
$$p(P) = \frac{1}{<P>} \exp\left(-\frac{P}{<P>}\right)$$

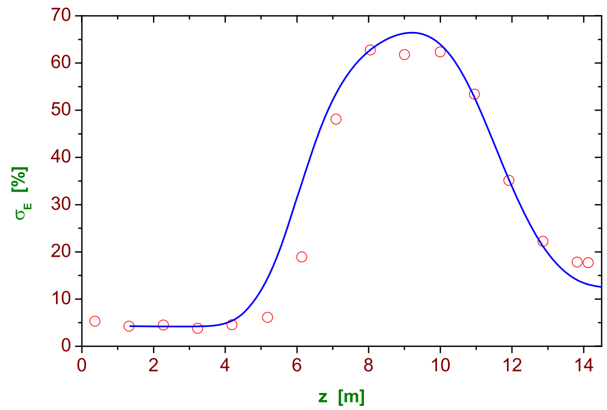
- The probability density function of the finite-time integrals of the instantaneous power and of the radiation energy after monochromator follows the gamma distribution

$$p(W) = \frac{M^M}{\Gamma(M)} \left(\frac{W}{<W>}\right)^{M-1} \frac{1}{<W>} \exp\left(-M\frac{W}{<W>}\right) , \qquad M^{-1} = \sigma_W^2 = <(W-<W>)^2> / <W>^2$$

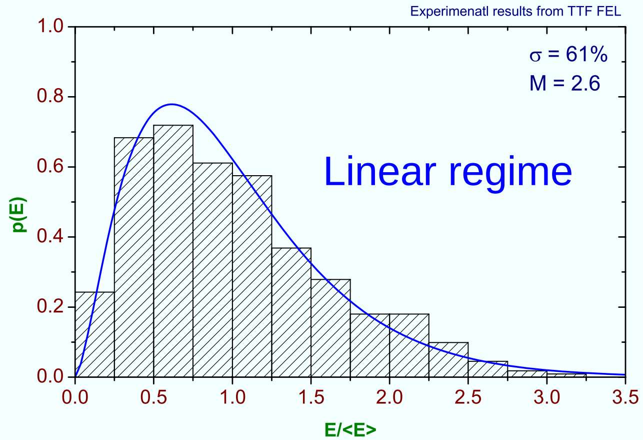


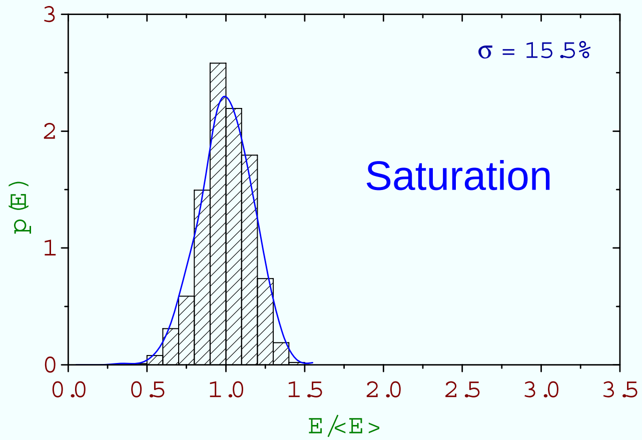
Statistics and probability distributions: Experimental results from TTF FEL/FLASH



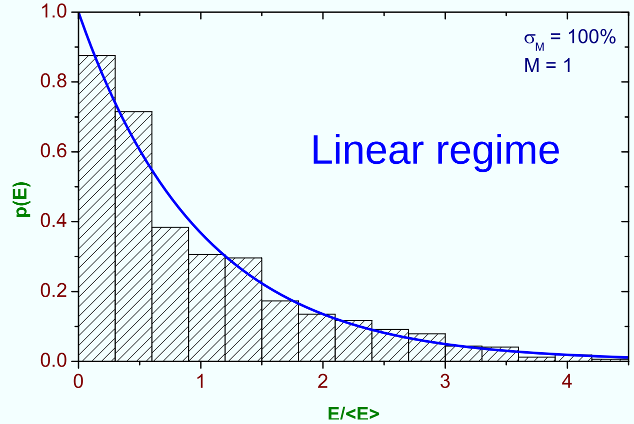


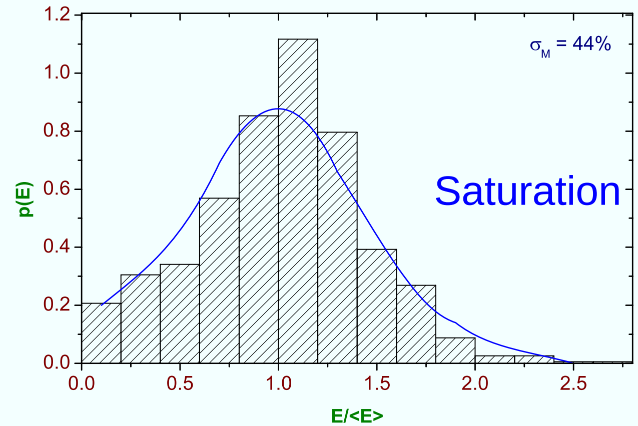
Probability distribution of the energy in the radiation pulse





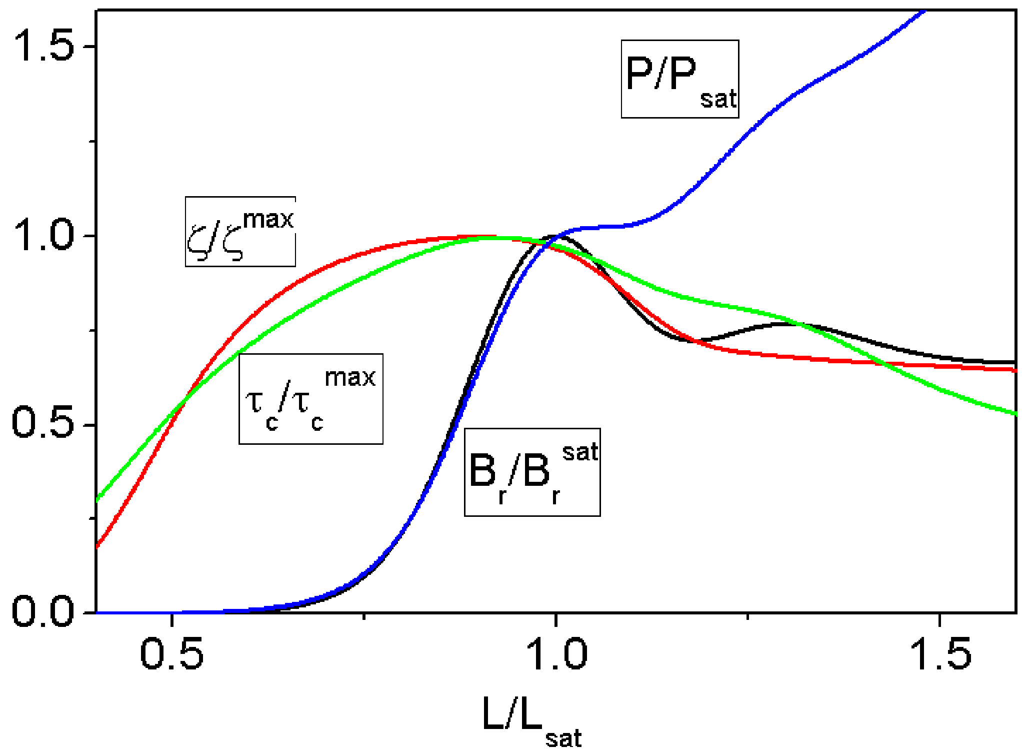
Probability distribution of the energy after narrow band monochromator





V. Ayvazyan et al., Nucl. Instrum. and Methods A 507 (2003)368

Qualitative look at the evolution of the radiation properties in XFEL



Radiation power

Brilliance

Degree of transverse coherence

Coherence time

- Radiation power continues to grow along the undulator length.
- Brilliance reaches maximum value at the saturation point.
- Degree of transverse coherence and coherence time reach their maximum values in the end of exponential regime.

Optimized XFEL

Radiation of XFEL operating in the high gain exponential regime can be presented as a set of self-reproducing radiation modes:

$$\tilde{E} = \int \mathrm{d}\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r,\omega)\exp[\Lambda_{nk}(\omega)z + in\phi] \ .$$

SASE FEL is optimized for maximum gain of fundamental (TEM_{00}) beam radiation mode. Gain and optimum beta function:

$$L_{\mathrm{g}} \simeq 1.67 \left(\frac{I_A}{I}\right)^{1/2} \frac{(\epsilon_n \lambda_{\mathrm{w}})^{5/6}}{\lambda^{2/3}} \frac{(1+K^2)^{1/3}}{K A_{JJ}} \ ,$$

$$\beta_{\mathrm{opt}} \simeq 11.2 \left(\frac{I_A}{I}\right)^{1/2} \frac{\epsilon_n^{3/2}\lambda_{\mathrm{w}}^{1/2}}{\lambda K A_{JJ}} \ .$$

Application of similarity techniques to the FEL equations gives elegant result: characteristics of SASE FEL written down in the normalized form are functions of two parameters, ratio of geomtrical emittance to the wavelength, and number of electrons in the volume of coherence:

$$\hat{\epsilon} = 2\pi\epsilon/\lambda \ , \qquad N_{\mathrm{c}} = I L_{\mathrm{g}} \lambda/(e\lambda_{\mathrm{w}} c) \ .$$

Dependence of the FEL characterestics on N_{c} is very slow, in fact, logarithmic. Approximately, with logarithmic accuracy they depend only on $\hat{\epsilon}$.

E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 235(2004)415, 281(2008)1179; 281(2008)4727.

Optimized XFEL at saturation

Saturation length:

$$\hat{L}_{\mathrm{sat}} = \Gamma L_{\mathrm{sat}} \simeq 2.5 \times \hat{\epsilon}^{5/6} \times \ln N_{\mathrm{c}} \ ,$$

FEL efficiency:

$$\hat{\eta} \ = \ P/(\bar{\rho} P_{\mathrm{b}}) \simeq 0.17/\hat{\epsilon} \ ,$$

Coherence time and rms spectrum width:

$$\hat{\tau}_{\mathrm{c}} \ = \ \bar{\rho}\omega\tau_{\mathrm{c}} \simeq 1.16 \times \sqrt{\ln N_{\mathrm{c}}} \times \hat{\epsilon}^{5/6} \ , \qquad \sigma_{\omega} \simeq \sqrt{\pi}/\tau_{\mathrm{c}} \ .$$

Degree of transverse coherence:

$$\zeta_{\mathrm{sat}} \ \simeq \ \frac{1.1\hat{\epsilon}^{1/4}}{1 + 0.15\hat{\epsilon}^{9/4}} \ ,$$

Degeneracy parameter:

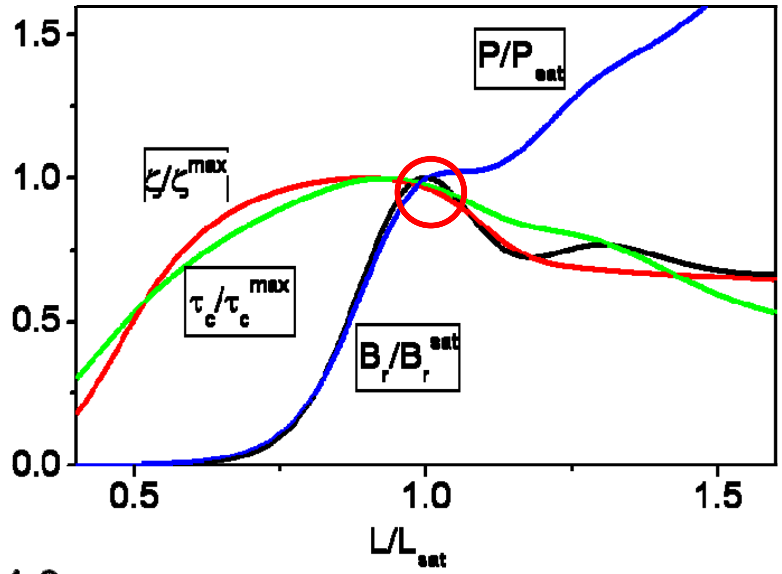
$$\hat{\delta} \ = \ \hat{\eta}\zeta\hat{\tau}_{\mathrm{c}}$$

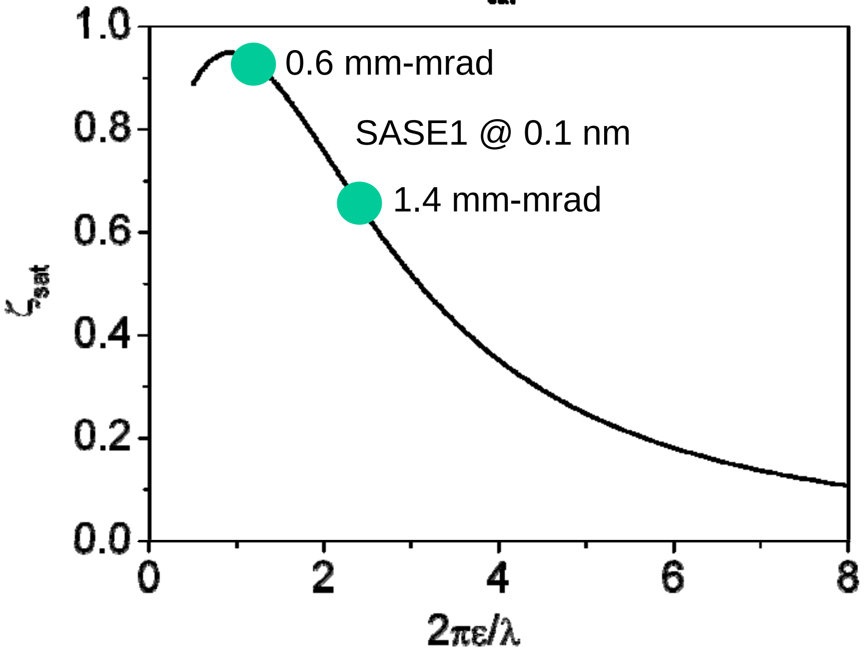
Brilliance:

$$B_r \ = \ \frac{\omega d\dot{N}_{ph}}{d\omega} \frac{\zeta}{\left(\frac{\lambda}{2}\right)^2} = \frac{4\sqrt{2}c}{\lambda^3} \frac{P_{\mathrm{b}}}{\hbar\omega^2}\hat{\delta} \ .$$

Normalizing parameters:

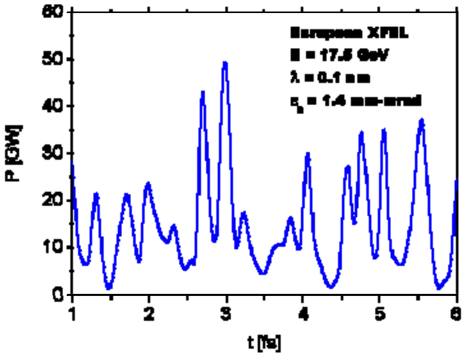
$$\Gamma \ = \ \left[\frac{I}{I_{\mathrm{A}}} \frac{8\pi^2 K^2 A_{\mathrm{JJ}}^2}{\lambda\lambda_{\mathrm{w}}\gamma^3}\right]^{1/2} \ , \qquad \bar{\rho} = \frac{\lambda_{\mathrm{w}}\Gamma}{4\pi} \ .$$

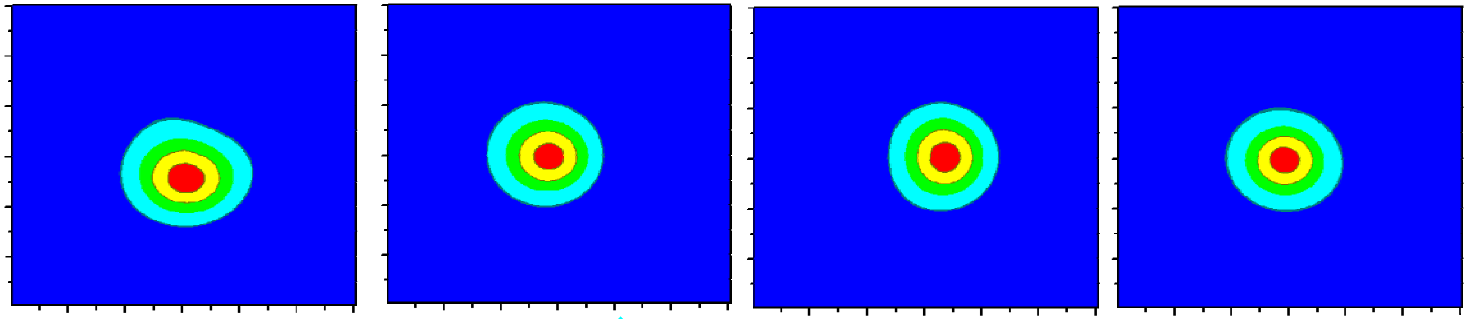




E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 281(2008)1179; 281(2008)4727 ; NJP, 2010 in press.

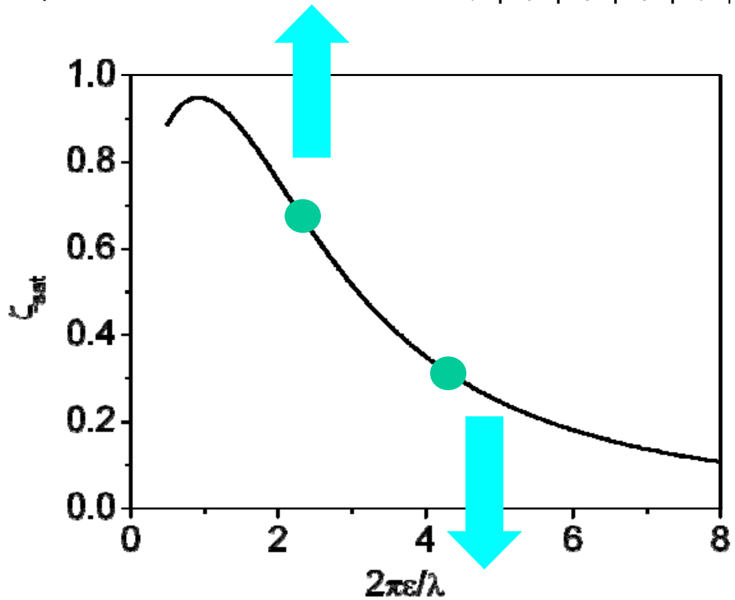
Qualitative look at the transverse coherence





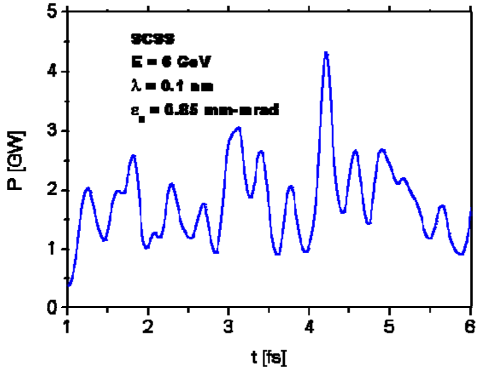
$2\pi\varepsilon/\lambda$ = 2.5, ζ = 0.65

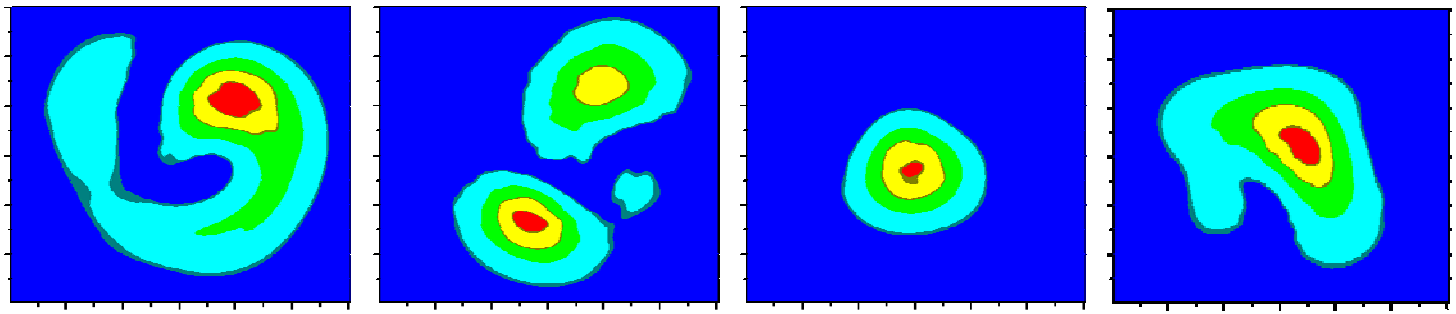
t = 2 fs, 2.7 fs, 3 fs, 4.1 fs



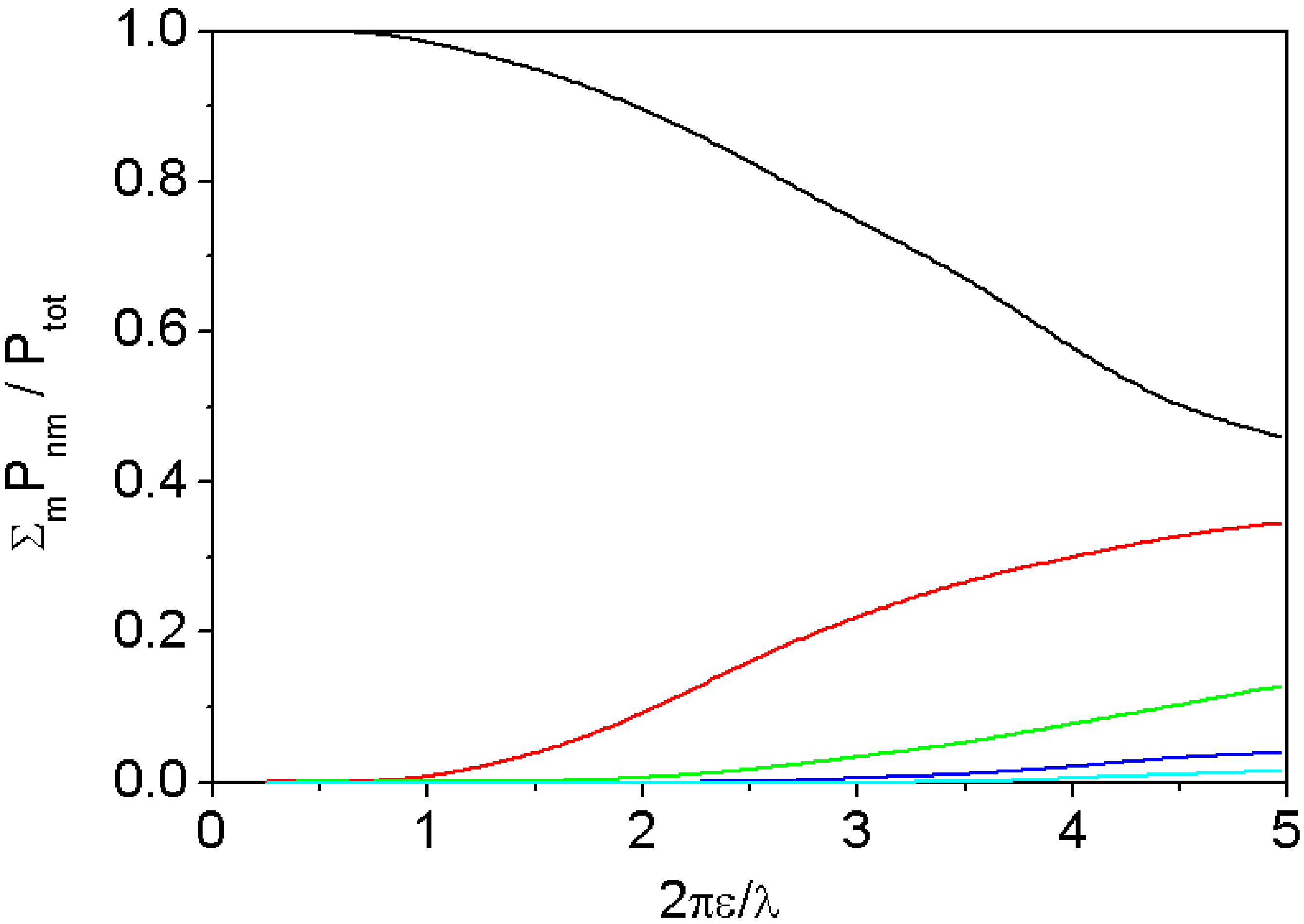
$2\pi\varepsilon/\lambda$ = 4.5, ζ = 0.4

t = 1.7 fs, 2.4 fs, 3.1 fs, 4.2 fs





Transverse coherence

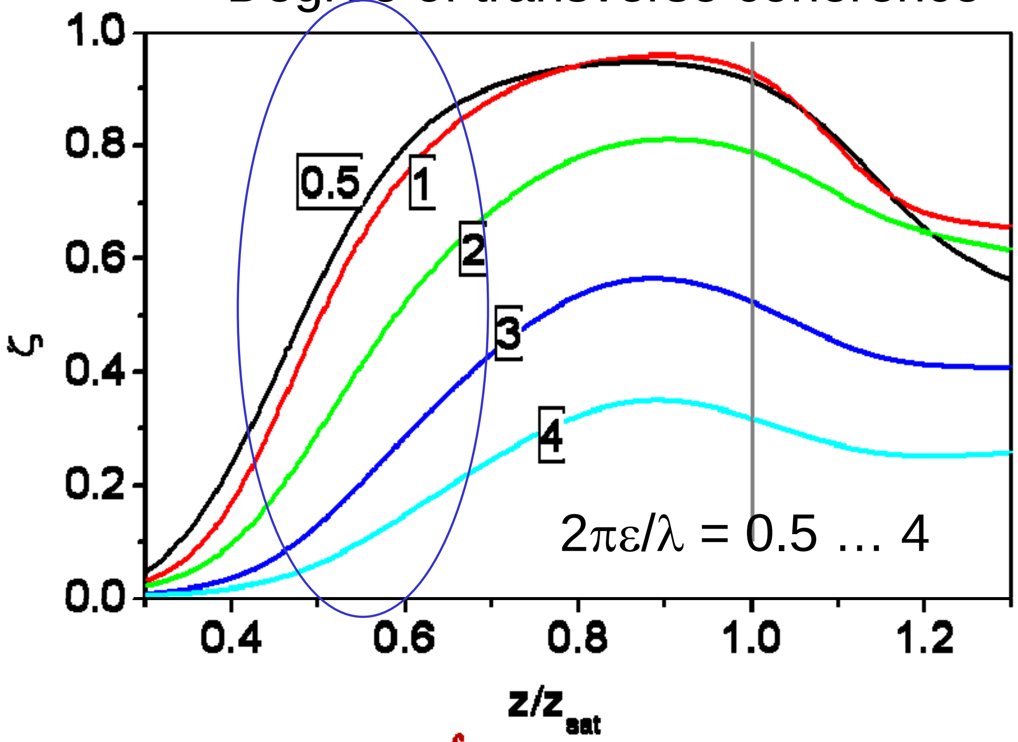


$$E_x + iE_y = \int d\omega \exp[i\omega(z/c-t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r,\omega)\exp[\Lambda_{nk}(\omega)z + in\phi]$$

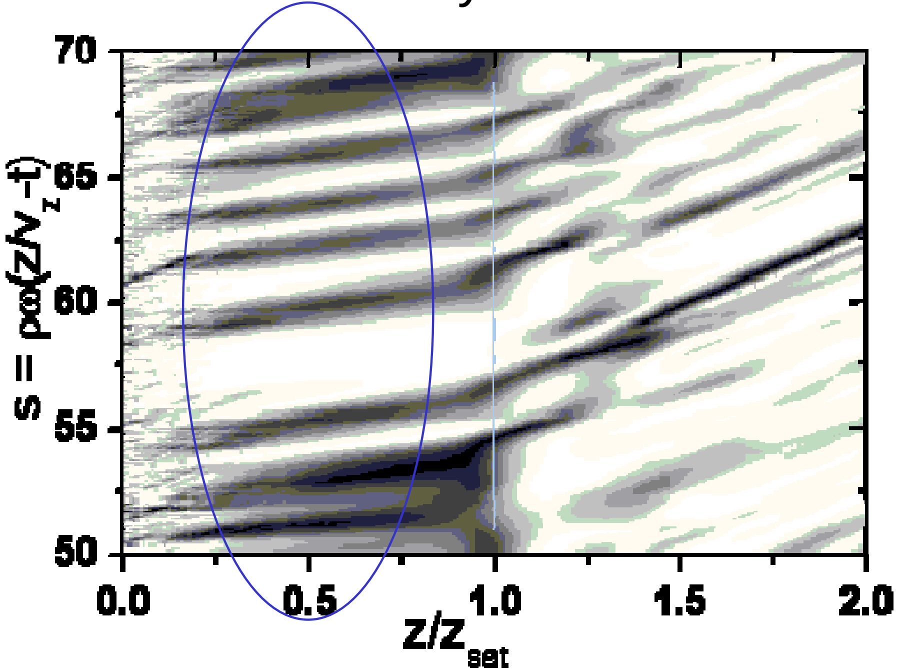
Contribution to the total saturation power of the radiation modes with higher azimuthal indexes 1, 2, 3, 4… grows with the emittance.

Transverse coherence

Degree of transverse coherence



z-s intensity distribution



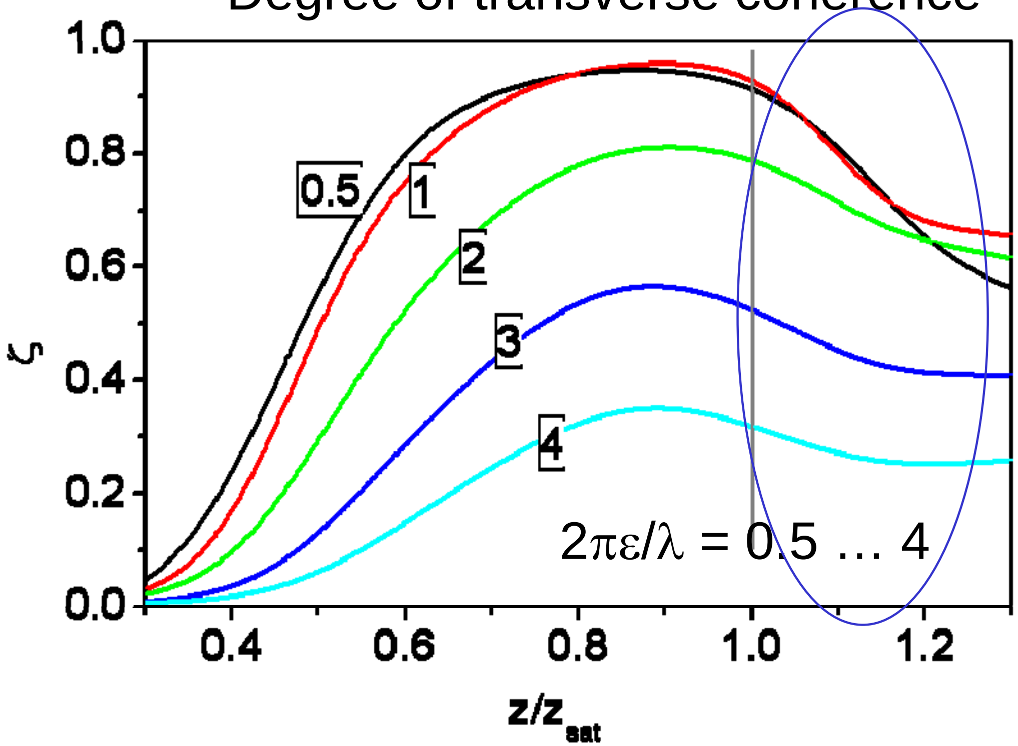
$$E_x + iE_y = \int d\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r,\omega) \exp[\Lambda_{nk}(\omega) z + in\phi]$$

- In the case of large emittance the degree of transverse coherence degrades due to poor mode selection.
- For small emittances the degree of transverse coherence visibly differs from unity. This happens due to poor longitudinal coherence: radiation spikes move forward along the electron beam, and interact with those parts of the beam which have different amplitude/phase.
- Longitudinal coherence develops slowly with the undulator length thus preventing full transverse coherence.

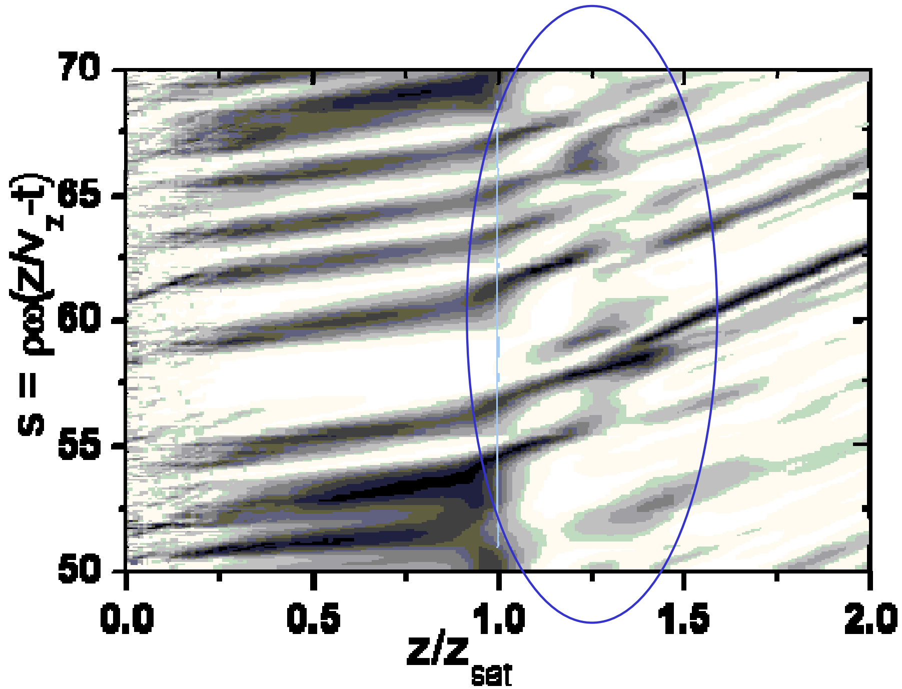
E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 186(2000)185

Transverse coherence

Degree of transverse coherence



z-s intensity distribution



- Poor longitudinal coherence is also responsible for the fast degradation of the transverse coherence in the nonlinear regime.
- In the linear exponential regime group velocity of spikes (/ ds/dz) is visibly less than the velocity of light due to strong interaction with the electron beam. In the nonlinear regime group velocity of spikes approaches velocity of light due to weak interaction with the electron beam.
- Radiation spikes move forward faster along the electron beam and start to interact with those parts of the beam which were formed due to interaction with different wavepackets.
- This process develops on the scale of the field gain length.

XFEL with planar undulator: odd harmonics

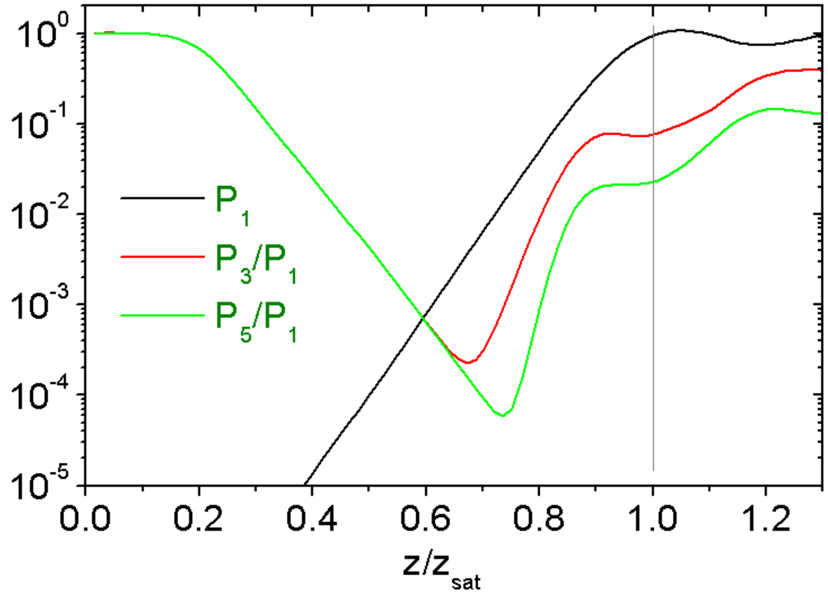
- In the saturation a universal dependency holds for the ratio of the power in the higher harmonics with respect to the fundamental one:

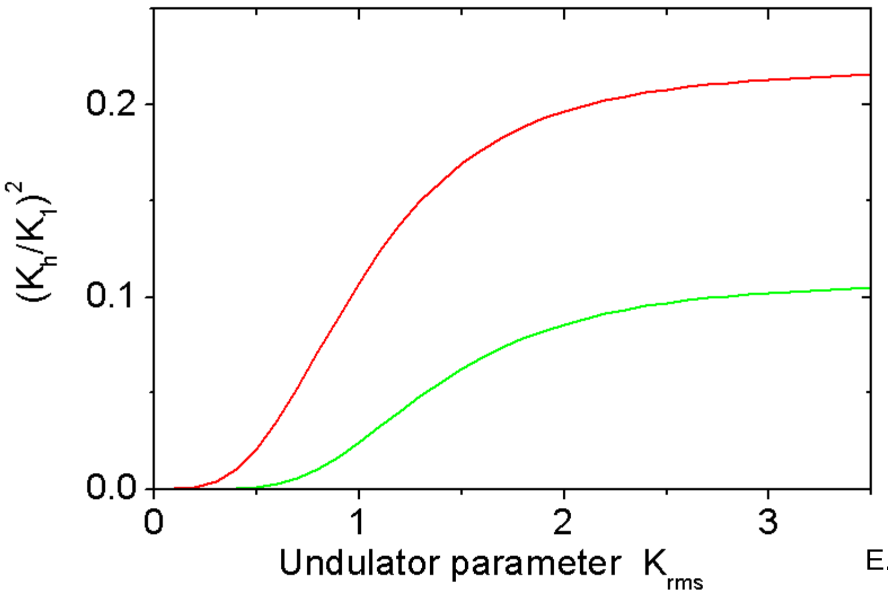
$$\frac{\langle W_3 \rangle}{\langle W_1 \rangle}\Big|_{\mathrm{sat}} = f_3(\hat{\Lambda}_{\mathrm{T}}^2) \times \frac{K_3^2}{K_1^2}\,,$$

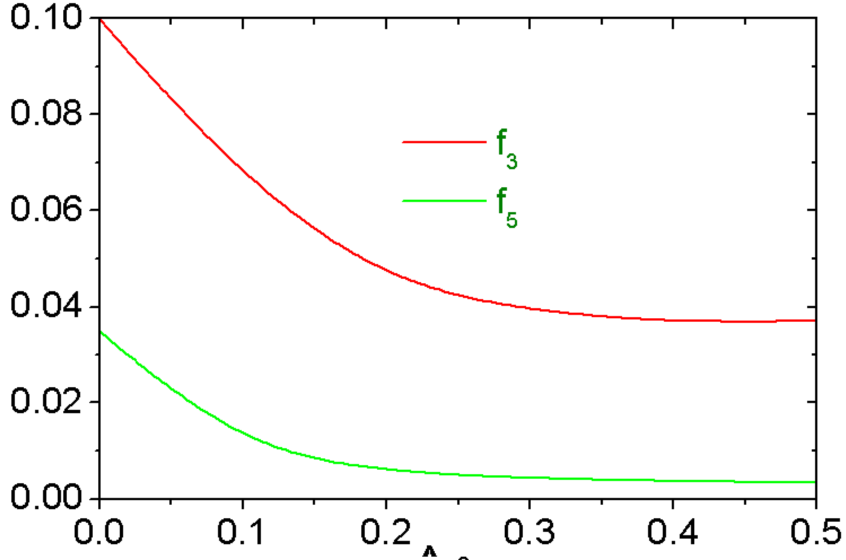
$$\frac{\langle W_5 \rangle}{\langle W_1 \rangle}\Big|_{\mathrm{sat}} = f_5(\hat{\Lambda}_{\mathrm{T}}^2) \times \frac{K_5^2}{K_1^2}\,.$$

$$K_h = K(-1)^{(h-1)/2}[J_{(h-1)/2}(Q) - J_{(h+1)/2}(Q)]$$

$$Q = K^2/[2(1+K^2)]$$







E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Phys. Rev. ST Accel. Beams 9(2006)030702

XFEL with planar undulator: odd harmonics

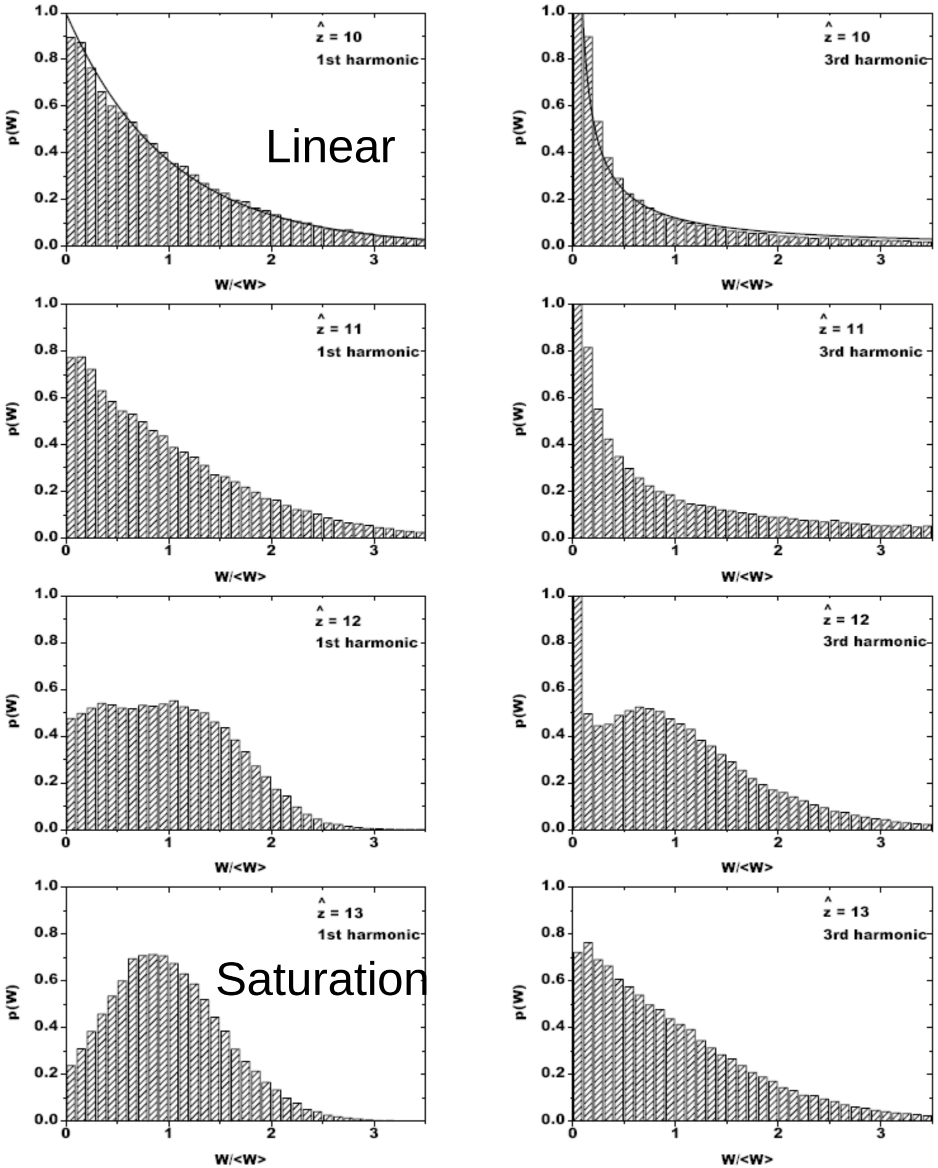
- The statistics of the high-harmonic radiation from the SASE FEL operating in the linear regime changes significantly with respect to the fundamental harmonic (e.g., with respect to Gaussian statistics).

- The probability density function of the fundamental intensity W:
$$p(W) = \langle W \rangle^{-1} \exp(-W/\langle W \rangle)$$
is subjected to a transformation $z = (W)^n$.

- The probability density function $p(z)$ is:
$$p(z) = \frac{z}{n\langle W \rangle} z^{(1-n)/n} \exp(-z^{1/n}/\langle W \rangle) \ .$$

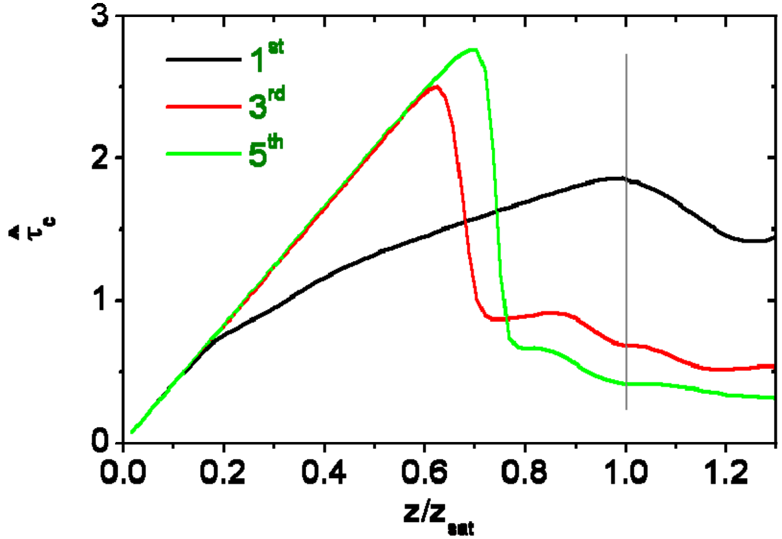
- Probability distribution of the instantaneous power of higher harmonics in saturation regime is close to the negative exponential distribution.

Evolution of probability distributions for the 1st and the 3rd harmonics

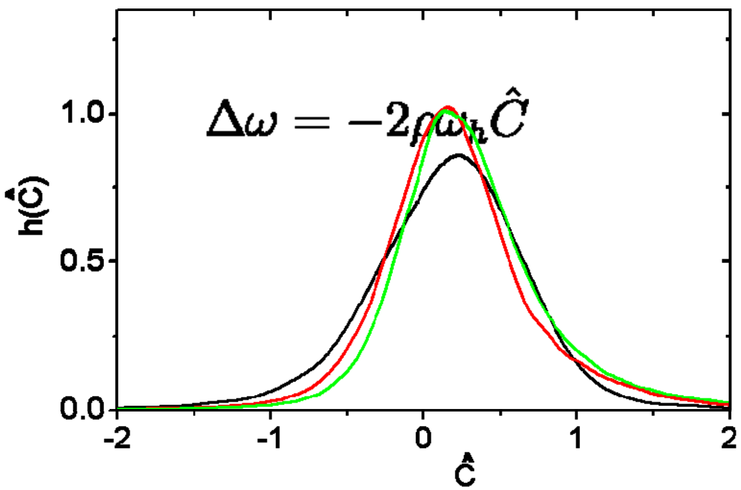


XFEL with planar undulator: odd harmonics

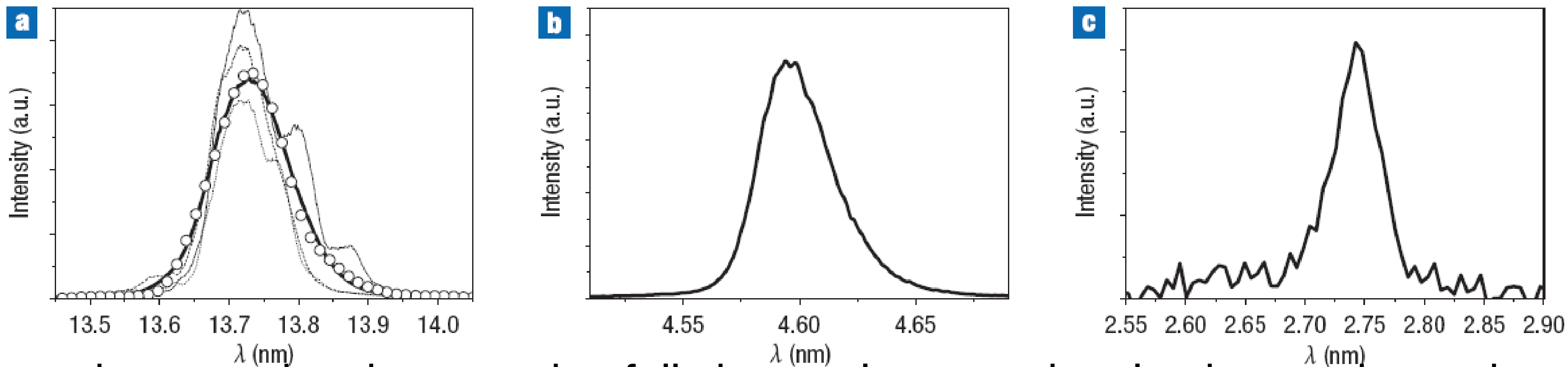
Coherence time: 1^{st}, 3^{rd}, 5^{th}



Average spectra: 1^{st}, 3^{rd}, 5^{th}



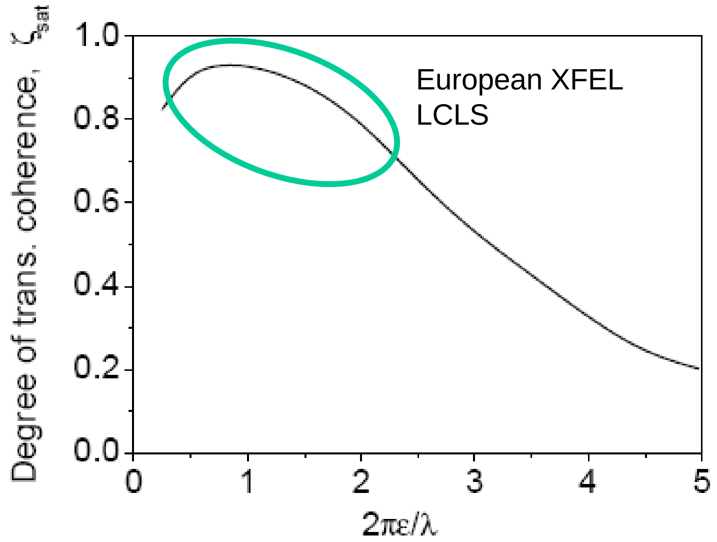
FLASH, 2006



- The coherence time in saturation falls inversely proportional to harmonic number.
- Relative spectrum bandwidth remains constant with harmonic number.

W. Ackermann et al., Nature Photonics, 1(2007)336

Summary of XFEL coherence



- Parameters of an optimized SASE FEL in the saturation are universal functions of the only parameter, $2\pi\varepsilon/\lambda$.
- The best transverse coherence properties are achieved for $2\pi\varepsilon/\lambda \sim 1$.
- At smaller values of the emittance the degree of transverse coherence is reduced due to strong influence of poor longitudinal coherence on a transverse one. At large values of the emittance the degree of transverse coherence degrades due to poor mode selection.
- XFEL driven by low energy (or, bad emittance) electron beam suffers from bad transverse coherence. Asymptotically degree of transverse coherence scales as

$$\zeta \simeq (\ln(N_c/\hat{\epsilon})/(4\hat{\epsilon}))^2 \ .$$

Thank you for your attention!

1D- Handbook: main properties of SASE FEL

- **FEL parameter ρ and number of cooperating electrons N_c:**

$$\rho = \frac{\lambda_w}{4\pi}\left[\frac{4\pi^2 j_0 K^2 A_{JJ}^2}{I_A \lambda_w \gamma^3}\right]^{1/3} , \quad N_c = I/(e\rho\omega) .$$

- **Main properties of SASE FEL in the saturation can be quickly estimated in terms of ρ and N_c:**

The field gain length : $L_g \sim \lambda_w/(4\pi\rho)$,

Saturation length : $L_{sat} \sim 10 \times L_g$,

Effective power of shot noise : $\frac{W_{sh}}{\rho W_b} \simeq \frac{3}{N_c\sqrt{\pi \ln N_c}}$,

Saturation efficiency : ρ ,

The power gain at saturation : $G \simeq \frac{1}{3} N_c \sqrt{\pi \ln N_c}$,

Coherence time at saturation : $\tau_c \simeq \frac{1}{\rho\omega}\sqrt{\frac{\pi \ln N_c}{18}}$,

Spectrum bandwidth : $\sigma_\omega \simeq \rho\omega\sqrt{\frac{18}{\ln N_c}}$.

- **In many cases this set of formulas can help quickly estimate main parameters of SASE FEL but it does not provide complete self-consistent basis for optimization of this device.**